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# Nobel numbers: Time-dependent centrality measures on co-authorship graphs 

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#### Abstract

A time-dependent centrality metric for disciplinary co-authorship graphs, the "Nobel number" for a discipline, is introduced. A researcher's Nobel number for a given discipline in a given year is defined as the researcher's average co-authorship distance to that discipline's Nobel laureates in that year. Plotting Nobel numbers over several decades provides a quantitative as well as visual indication of a researcher's proximity to the intuitive "center" of a discipline as defined by recognized scientific achievement. It is shown that the Nobel number distributions for physics of several researchers both within and outside of physics are surprisingly flat over the five-decade span from 1951 to 2000. A model in which Nobel laureates are typically connected by short co-authorship paths both intergenerationally and between subdisciplines reproduces such flat Nobel number distributions.


Keywords: Betweenness centrality; Biomedicine; Cross-disciplinary brokerage; Erdős numbers; Interdisciplinarity; Physics

## Introduction

What does it mean to say that a researcher is "central" to a discipline? In particular, what does it mean to say that a researcher is central to the social network represented by the discipline's co-authorship graph? Formal, graph-theoretic measures of centrality provide some candidate answers. One can identify the researchers with the largest numbers of co-authors, who have "degree centrality" in graphtheoretic terms (for definitions of relevant terms and concepts of graph theory, see Börner, Sanyal and Vespignani, 2007; Diestel, 2010). Alternatively, one can calculate the length of longest minimal path between any two researchers; this length is the "diameter" of the co-authorship graph. Any researcher
separated by no more than half of the graph diameter from any other researcher has "distance centrality" within the graph. A third option is to identify those researchers through whom the largest numbers of minimal paths connecting other researchers flow; they have "betweenness centrality." These three formal measures of centrality do not pick out the same researchers as "central" unless the co-authorship graph is highly symmetrical (see Fields, 2014, Fig. 2 for an example graph in which the three measures coincide and another in which they do not), and their meaningfulness as measures of centrality in co-authorship graphs remains a topic of considerable debate (e.g. Freeman, 1978/79; Borgatti and Everett, 2006; Landherr, Friedl and Heidemann, 2010). Minor members of large collaborations, for example, may have many co-authors and hence high degree centrality, otherwiseunremarkable "weak links" between subdisciplines may have high betweenness centrality, and distance centrality reflects academic prominence within a discipline only if the most prominent individuals, and no others, occupy the metric center of the discipline's co-authorship graph.

Intuitively and informally, the centrality of a researcher within a discipline is not determined by his or her position in the co-authorship graph, but rather by relevant scientific accomplishments. Wellknown, well-respected intellectual leaders, who are often also well-funded, highly-productive political leaders, are widely acknowledged to be "central" players in any discipline. The recognitions and honors that disciplines bestow, such as Nobel Prizes, Fields Medals or Abel Prizes in mathematics, the Catherine Wolfe Bruce Gold Medals in astronomy or A. M. Turing Awards or John von Neumann Medals in computer science, are generally awarded to such recognized leaders. It seems reasonable, therefore, to regard "centrality" within a co-authorship graph not as a formal, graph-theoretic notion but rather as informally defined by recognized scientific accomplishments, and to seek metrics for coauthorship centrality that honor this informal definition.

As Nobel prizes are awarded annually, the Nobel laureates of a discipline provide a time-dependent marker of the informal "center" of that discipline as defined by recognized scientific achievement. Note that "recognized" here means recognized by the award of a Nobel prize, not recognized at the time that the relevant work was done or published, a discrepancy perhaps most notable in the case of Barbara McClintock (Physiology or Medicine, 1983), ${ }^{1}$ whose work was largely dismissed when first reported (Keller, 1983). Research work that is regarded as "central" at the time that a Nobel prize is awarded may, in other words, not have been regarded as "central" or even as valid when the work was performed; similarly, researchers who are regarded as "central" when a Nobel prize is awarded may have been regarded as marginal when they performed their Nobel-prize-winning work.

Here I propose a time-dependent centrality metric for disciplinary co-authorship graphs, the "Nobel number" for a discipline, that reflects this informal, community-recognition-based notion of centrality. Nobel numbers measure the average distance in co-authorship units - i.e. the average of the minimal co-authorship path lengths - from any researcher to that discipline's Nobel laureates for a given year. Metrics based on other community-awarded prizes, such as Fields Medals or Turing Awards, can be analogously defined. To illustrate the utility and informativeness of this kind of metric, I consider the Nobel laureates in Physics in the latter half of the $20^{\text {th }}$ century, i.e. from 1951 to 2000. This is the period during which high-energy physics emerged as a distinct and financially dominant subdiscipline within physics; it also saw the development of high-speed computers and "big science" collaborations, the emergence of solid-state physics, laser physics, observational astrophysics and quantum

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Nobel laureates are indicated throughout by an award date. The award category is included for laureates in disciplines other than physics.
information theory, and technological developments including space travel, precision time measurement and the ability to manipulate single atoms. The time-dependent Nobel number $N_{P}(t)$ of any researcher with respect to this collection of 108 Nobel laureates in Physics would be expected to provide a natural, time-dependent measure of his or her distance from the recognized intellectual "center" of physics during this period of rapid growth and development.

One might expect the Nobel number distribution for any individual with respect to any discipline awarding Nobel prizes to exhibit significant temporal structure, e.g. to have broad maxima and minima over the time range examined that correlate with the individual's subdiscipline, academic lineage, or some other variable(s). I show that, at least for physics during the time period examined, this does not appear to be the case. Even the Nobel number distributions for physics of biomedical researchers here, Nobel laureates in Physiology of Medicine from 1991 to 2010 - are remarkably flat. I interpret this somewhat surprising result in terms of the high degree of both intergenerational and crosssubdisciplinary co-authorship connections among Nobel laureates in physics that is evident even in small samples from the complete co-authorship graph.

## Methods and Data

Names and specializations of all Nobel laureates in Physics from 1951 to 2000 were obtained from Nobelprize.org. ${ }^{2}$ Co-authorship data were obtained from Google Scholar ${ }^{\mathrm{TM}}$ as described (Fields, 2015 a); co-authorship data were not filtered or otherwise restricted on the basis of the date of the coauthored publication. Paths from laureates traversing other authors known to be near either crossdisciplinary brokers or other Nobel laureates were followed preferentially. This is a tractable, greedy search procedure that produces upper limits on the minimal co-authorship path lengths from laureates to either other laureates or brokers; more exhaustive, and in particular non-heuristic, search procedures may produce smaller path lengths in some cases. "Cross-disciplinary brokers" were defined as individuals who have published co-authored papers both in physics and in at least one of the 15 other Klavans and Boyack (2009) consensus disciplines. Disciplinary assignments of papers were determined from the title or abstract, or if necessary, by reading the paper in its entirety. Only primary and secondary research papers, review articles, research-based science-policy papers and scholarly books were included in the co-authorship analysis; otherwise-unpublished technical reports, textbooks, joint editing of collections, and editorial or opinion pieces were not included. Where necessary, authors with similar names were disambiguated by tracing their histories of institutional appointments.

The time-dependent Nobel number $N_{P}(t)$ for physics of a given researcher is defined as the average, for the year $t$, of the minimal co-authorship path lengths from that researcher to that year's Nobel laureates in Physics. The time-dependent Nobel number $N_{M}(t)$ for biomedicine is defined similarly, for each year's Nobel laureates in Physiology or Medicine. All values of $N_{P}(t)$ or $N_{M}(t)$ quoted or displayed here were computed from upper limits on minimal co-authorship path lengths as described above and therefore must be considered upper limits; optimal search procedures may produce smaller values in some cases.

Random numbers for the statistical model described in the Discussion below were generated using the javascript Math.random function.

2http://www.nobelprize.org/nobel prizes/physics/laureates/; accessed Sept. - Dec., 2014.

## Results

Representing the "center" of $20^{\text {th }}$ century physics
Physics in the $20^{\text {th }}$ century was largely devoted to the development, experimental testing and technological implications of three novel theories, the special and general theories of relativity and quantum theory. From the 1901 award to Wilhelm Röntgen for his discovery of X-rays to the honoring of Zhores Alferov, Herbert Kroemer and Jack Kilby in 2000 for their work on semiconductors, $20^{\text {th }}$ century Nobel prizes in Physics reflect this focus. The $20^{\text {th }}$ century Nobel laureates in Physics form, moreover, a remarkably close multi-generational family. One view of this family is shown in Fig. 1, which displays co-authorship connections between 34 Nobel laureates in Physics from Albert Einstein (1921) to Frederick Reines (1995) and 9 of their collaborators. As noted, the co-authorship links are not restricted by date; pairs of authors may have published together either before or after one or both received Nobel prizes. The founders of quantum theory are well represented in Fig. 1: Einstein, Niels Bohr (1922), Louis de Broglie (1929), Werner Heisenberg (1932), Paul Dirac (1933), Erwin Schrödinger (1933), Wolfgang Pauli (1945), Max Born (1954) and Eugene Wigner (1963). Several leading participants in the Manhattan Project are also included: Enrico Fermi (1938), Isidor Rabi (1944), Emilio Segrè (1959), Richard Feynman (1965), Hans Bethe (1967), Norman Ramsey (1989) and Frederick Reines. James Chadwick (1935) discovered the neutron, Frederick Reines discovered the neutrino, Chen Ning Yang (1957), Tsung-Dao Lee (1957) and Murray Gell-Mann (1969) developed key concepts of the theory of elementary particles, and Samuel Ting (1976), Carlo Rubbia (1984) and George Charpak (1992) were pioneers of experimental high-energy physics. The co-authorship network shown in Fig. 1 provides, therefore, a dense sample from the "center" of physics as intuitively defined on the basis of recognized scientific accomplishment and influence in the $20^{\text {th }}$ century.


Fig. 1: A sample from the "center" of the co-authorship graph of $20^{\text {th }}$ century physics as defined by Nobel laureates. Note the large distance from Albert Einstein (Nobel Prize in Physics, 1921) to Niels Bohr (Nobel Prize in Physics, 1922), with whom Einstein famously disagreed about the interpretation of quantum theory.

It is natural, however, to ask whether Fig. 1 or indeed any partial sample of the co-authorship graph of a discipline accurately represents a disciplinary center. Nobel numbers provide one approach to answering this question. The Nobel numbers for physics of two prominent physicists, nuclear theorist and founding Director of Lawrence Livermore National Laboratory (LLNL) Edward Teller and condensed-matter and complex systems theorist David Pines, are shown in Fig. 2. Teller is a direct coauthor of three Nobel laureates and appears near the center of Fig. 1. Pines has co-authored papers with seven Nobel laureates and is within a few steps of many more (De Castro and Grossman, 1999); his closest connections are summarized in Fig. 3. Teller and Pines are separated by three co-authorship steps along multiple paths; as can be seen in Fig. 2, however, their values for $N_{P}(t)$ differ by this much only in only four years: 1956 and 1972, when Pines' co-author John Bardeen won Nobel Prizes, and 1985 and 2000. Teller's 50-year average $\left.<N_{P}\right\rangle$ is 4.8; Pines' is 4.9. Newman (2001) calculated an average co-authorship distance of 5.9 between physicists submitting papers to the arXiv preprint database, which spans subdisciplines of physics, in the latter half of the 1990s; both Teller and Pines were, therefore, closer to Nobel laureates in Physics, on average, in the second half of the $20^{\text {th }}$ century than physicists in general were, again on average, to each other. Teller and Pines were both distant from Nobel laureates only in 1986, 1987 and 2000, when prizes were awarded for technical work in microscopy, ceramics and semiconductors respectively.


Fig. 2: Nobel numbers $N_{P}(1951)-N_{P}(2000)$ for two prominent physicists, Edward Teller and David Pines. The dashed horizontal line shows their combined 50-year average Nobel number for physics of 4.85 .


Fig. 3: Close co-authorship connections of condensed-matter and complexity theorist David Pines to Nobel laureates in Physics. Compare with De Castro and Grossman (1999), Fig. 1, a previous study of Pines' many connections.

Figure 2 can also, clearly, be viewed as displaying the average distances of each year's Nobel laureates to a "center" defined by Teller and Pines. This alternative view is explored further in Fig. 4a, which shows the average distances $d_{C}(t)$ of each year's Nobel laureates to the multi-generational "center" shown in Fig. 1, defined for each year as the average of the distances of that year's Nobel laureates to the closest node in Fig. 1 for each of the laureates. As can be seen, the average values of $d_{C}(t)$ almost double during this time period, from a 10-year average of 2.1 in the 1950s to a 10-year average of 3.7 in the 1990s. This increase in average distance from Fig. 1 reflects a gradual change in the kinds of work for which Nobel prizes were awarded. In the decade from 1951 to 1960, two Nobel prizes (1953 and 1956) honored work outside of the "mainstream" of atomic, nuclear and particle physics that is represented by Fig. 1. In the 1981 to 1990 decade, however, seven Nobel prizes were awarded for work outside of this area, while in the 1991 to 2000 decade, six Nobel prizes were awarded for work outside of this area. The main emphasis of Nobel prizes in Physics, and hence the intuitive "center" of
physics as a discipline, thus shifted gradually as the $20^{\text {th }}$ century drew to a close. The greatest average distance from Fig. 1 is in 1986, when the Nobel prize in Physics honored Gerd Binnig, Heinrich Rohrer and Ernst Ruska's development of microscopy technologies that were initially applied in biology, not physics.


Fig. 4: a) Average co-authorship distances $d_{C}$, by award year, between Nobel laureates and the closest node of Fig. 1. b) Greatest inter-laureate co-authorship distances $\Delta$, again by award year.

Despite this shift in the subdisciplinary emphasis of Nobel prizes, however, the co-authorship network shown in Fig. 1 remained near the "center" of late $20^{\text {th }}$ century physics as defined by co-authorship betweenness centrality. Maximal upper-limit co-authorship distances $\Delta(t)$ between Nobel laureates in each year $t$ are shown, by award year, in Fig. 4b. Large inter-laureate distances in the 1950s and early 1960s mainly reflect the relatively low levels of co-authorship prior to World War II; the $\Delta=10$ in 1964 reflects the separation of Charles H. Townes from Nicolay G. Basov and Aleksandr M. Prokhorov by the "Iron Curtain" then in place between the NATO countries and the USSR. Beginning in 1970, however, large values of $\Delta$ tend to reflect distances between subdisciplines. The 1973 Nobel prize, for example, honored work in semiconductors and superconductors, two quite different areas of research; the value of $\Delta=11$ between laureates Leo Esaki and Ivar Giaever reflects this difference. The average distance of these researchers to Fig. 1, however, is only $d_{\mathrm{C}}=5$; hence Fig. 1 is between them. The situation in $1978(\Delta=12)$ is similar; Arno Penzias and Robert Wilson were honored that year for work in astrophysics, while Pyotr Kapitsa's share of the prize honored his work in cryogenics. In 1994, the Nobel prize was divided between Bertram Blockhouse, a nuclear chemist, and Clifford Shull, an atomic physicist. The largest separation observed here, $\Delta=18$ in 2000, separates scientists who spent virtually their entire careers in industry and had relatively few co-authors. For comparison, Newman (2001) reports a co-authorship diameter of 20 for the arXiv database, a reasonable surrogate for the academic physics literature, at this time. Even in this extreme case, the closest co-authorship path found that connects Jack Kilby to his co-laureates Zhores Alferov and Herbert Kroemer traverses Fig. 1, confirming its betweenness centrality.

## Nobel numbers between disciplines

While Nobel numbers provide a potentially interesting representation of centrality within a discipline as shown above, they are perhaps more interesting as a time-dependent measure of the co-authorship distances between disciplines. Paul Erdős' Nobel numbers - i.e. the average Erdős numbers, by year, of Nobel laureates - provide a natural test case. As shown in Fig. 5, Erdős' values of $N_{P}(t)$ remain remarkably stable from 1951 to 2000 (co-authorship paths to Erdős for all Nobel laureates in Physics from 1951 to 2000 are provided in the Appendix). Compared to Newman's (2001) average coauthorship distance of 5.9 for physicists, Erdős' 50 -year average $\left\langle N_{P}\right\rangle$ of 5.7 indicates that Nobel laureates in Physics were as close, on average, to Erdős and hence to discrete mathematics as physicists in general were, on average, to each other. It is also interesting that Erdős, a mathematician, is on average only one co-authorship step farther from Nobel laureates in Physics during this period than are Edward Teller and David Pines, two clearly "central" physicists.


Fig. 5: Nobel numbers $N_{P}(1951)-N_{P}(2000)$ for mathematician Paul Erdős, for whom Erdős numbers are named. Erdős' values of $N_{P}$ are the averages, by year, of the Erdős numbers of the laureates (see the Appendix for a complete list).

Physics has traditionally been a mathematical discipline, so it is perhaps not surprising that the "center" of physics as defined by Nobel laureates is close to mathematics. Of the scientific disciplines in which Nobel prizes are awarded, biomedicine is perhaps the most distant, intuitively, from physics. Chen, Arsenault, Gingras and Larivière (2014) showed by analyzing cross-disciplinary citations from 1910 to 2012 that chemical physics began to influence biomedical research in the early 1960s (see their Fig. 8); they note no additional significant influences of physics on biomedicine. Using a combination of termfrequency and citation analyses, Waltman, van Raan and Smart (2014) demonstrate a robust interaction between physical (including engineering) and biomedical sciences during the decade 2001-2010, noting in particular the rapid growth of medical statistics and informatics and its correlation with the development of proteomics and metabolomics, both of which present substantial data analysis challenges, as research areas. The one-decade timeframe of this latter study does not, however, permit a straightforward comparison with the historical analysis of cross-disciplinary interactions presented by Chen et al. (2014).

Collaboration and co-authorship provide routes for cross-disciplinary influences that may not be adequately reflected in citation counts. The labeling of nucleic acids with radioactive ${ }^{32} \mathrm{P}$, which until its replacement by fluorescent labeling in the 1990s was standard practice in molecular biology laboratories (e.g. Maniatis, Fritsch and Sambrook, 1982), provides a case in point. The use of ${ }^{32} \mathrm{P}$ to
label biologically-active compounds was introduced by Chievitz and Hevesy (1935). Otto Chievitz was a physiologist; George Hevesy (Chemistry, 1943) was a nuclear physicist, a colleague though not a co-author of Niels Bohr at the Institute for Theoretical Physics in Copenhagen. Chievitz and Hevesy (1935) includes an acknowledgement to Bohr but no references; in particular, it includes no references to the physics of radioactivity, the methods for detecting it, or the production methods or half-life measurement of ${ }^{32} \mathrm{P}$. This lack of citations to the relevant physics literature is not unusual. Perlman, Rubin and Chaikoff (1937), for example, acknowledge the assistance of E. O. Lawrence and diagram the nuclear reaction used to produce ${ }^{32}$ P, but cite no physics papers; Cook, Scott and Abelson (1937) similarly thank Lawrence but include no physics citations. The experiments with ${ }^{32} \mathrm{P}$ that Chievitz and Hevesy (1935) reported, together with subsequent studies by Hevesy and others, laid the groundwork for sizing and later sequencing DNA using ${ }^{32} \mathrm{P}$ as a label (e.g. Southern, 1975; Sanger, Nicklen and Coulson, 1977); a search of Google Scholar ${ }^{\mathrm{TM}}$ with the phrase " ${ }^{\text {"32 }} \mathrm{P}$ DNA" yields 161,000 results. However, only 191 publications listed in Google Scholar ${ }^{\mathrm{TM}}$ cite Chievitz and Hevesy (1935), despite its being considered worthy of a celebratory $40^{\text {th }}$ anniversary reprinting in the Journal of Nuclear Medicine (Vol. 16, pp. 1106-1107). Contemporaneous review articles introducing the new radiolabeling techniques to biologists fared considerably worse; Lawrence (1937), which cites some of the relevant physics literature (the author was E. O. Lawrence's brother), has received 11 citations to date; Krogh (1937), which cites no references at all, has received $14 .^{3}$

Atomic physicist Max Delbrück, a student of Bohr in Copenhagen, was one of the founders of molecular biology and won a Nobel Prize in Physiology or Medicine (1969) as a result, but had ceased active research in physics by that time. Physics laureate Richard Feynman (1965) co-authored a paper in molecular biology (Edgar et al., 1962), but did not pursue it further. Several physicists were, however, directly involved in the Human Genome project, and Nobel laureates in Physiology or Medicine between 1991 and 2010 are as close to physics, on average, as they are to mathematics (Fields, 2015a). The average Erdős number of these Nobel laureates in Physiology or Medicine, and hence Erdős' average Nobel number $\left\langle N_{M}\right\rangle$ for biomedicine during this period, is 5.5 (Fields, 2015a). Hence Nobel laureates in Physics from 1951 to 2000 are separated from Nobel laureates in Physiology or Medicine from 1991 to 2010, on average, by at most $5.5+5.7=11.2$ co-authorship steps through Erdős, i.e. by paths of average length about 11 that traverse Erdős' subdiscipline of discrete mathematics.

This average cross-disciplinary inter-laureate distance can be expected to decrease if direct coauthorship connections between physics and the biomedical sciences are considered. My own career started in physics and continued in bioinformatics; hence I am a cross-disciplinary broker between physics and biomedicine. Figure 6 compares my Nobel numbers $N_{P}(t)$ and $N_{M}(t)$ for 1951-2000 and 1991-2010 respectively. On average, I am 5.8 co-authorship steps from Physics laureates and 4.1 coauthorship steps from Physiology or Medicine laureates during these periods; hence the two groups of Nobel laureates are separated, on average, by a distance of no more than 10 co-authorship steps on paths that traverse me. This upper limit on average cross-disciplinary separation is half of the coauthorship graph diameter of 20 for physicists, i.e. half of the distance between the most-distant physicists, and less than half of the co-authorship graph diameter of 24 for biomedical scientists (Newman, 2001) during the relevant time periods. Hence if these Nobel laureates are central to their disciplines, disciplinary centrality cannot be captured by distance centrality in the disciplinary coauthorship graphs of physics and biomedicine during the time periods examined. This outcome, which

[^0]is illustrated schematically in Fig. 7, is consistent with the existence of multiple Nobel laureates who are themselves cross-disciplinary brokers (Fields, 2015a; 2015b).


Fig. 6: Nobel numbers $N_{P}(1951)-N_{P}(2000)$ for physics (upper part of graph) of physics-to-biomedicine cross-disciplinary broker C. A. Fields compared to Nobel numbers $N_{M}(1991)-N_{M}(2010)$ for biomedicine (lower part of graph).


Fig. 7: Schematic comparison of the co-authorship graphs of physics and biomedicine, showing the graph diameters $D_{\text {physics }}$ and $D_{\text {biomedicine }}$ reported by Newman (2001) and the average distance $<d>$ between Nobel laureates computed on paths traversing cross-disciplinary broker C. A. Fields. The shaded circles representing the typical locations of Nobel laureates within each discipline are shown in contact with the boundary between the disciplines to indicate that some Nobel laureates, e.g. R. P. Feynman (Physics, 1965) and Max Delbrück (Physiology or Medicine, 1969), are themselves physics-to-biomedicine brokers.

It is reasonable to expect upper-limit distances between Nobel laureates from different disciplines to decrease as paths traversing additional cross-disciplinary brokers are investigated. Paths of length 4 or 5 between Physiology and Medicine laureates and Physics laureates that traverse either myself or my bioinformatics colleague Eric Lander, currently Director of the Broad Institute and hence himself a central figure in biomedicine under any reasonable definition, are shown in Fig. 8. Lander and I share paths of length two to five Nobel laureates in Physiology and Medicine, but our paths into physics are disjoint at length three. Hence we differ in betweenness centrality as a function of the Nobel laureates connected, showing the advantage of using co-authorship data for multiple cross-disciplinary brokers to estimate minimal paths between disciplinary centers.


Fig. 8: A sample from the border between physics and the biomedical sciences, showing co-authorship paths of length 4 or 5 between Nobel laureates in Physics (top half of graph) and Physiology or Medicine (bottom half of graph) that traverse either C. A. Fields or E. S. Lander. Note that paths through Lander also traverse mathematician Daniel Kleitman, whose Erdős number is one. Several of the Physics laureates shown also appear in Fig. 1; for relations between the Physiology or Medicine laureates, see Fields (2015a). It is interesting that A. V. Carrano, G. N. Cox and R. S. Edgar are all biologists.

## Discussion

The distributions of Nobel numbers for individual scientists shown in Figs. 2, 5 and 6 share an interesting feature: not only are their decade-to-decade mean values approximately constant, but their decade-to-decade dispersions from the mean are approximately constant as well. Such flat distributions are prima facie surprising. One might expect, for example, that nuclear physicists would be close to Nobel laureates in nuclear physics and distant from Nobel laureates in other areas, or close to Nobel laureates during some particular one or two decades and more distant from those of other decades. As noted above, Fig. 4a displays such time dependence; the researchers included in Fig. 1 are mostly nuclear and particle physicists and they are closer, as a group, to nuclear and particle physicists than they are to condensed-matter physicists, biophysicists or astrophysicists. This does not, however, appear to be the case for individuals, regardless of their subdisciplinary specializations.

My own Nobel number distributions provide a somewhat extreme case in point. My connections to Physics laureates, with the exceptions of Glaser and Feynman, all result from publications in experimental nuclear physics during the 5-year period 1979-1983, while my connections to Physiology or Medicine laureates, as well as to Glaser, all result from publications in genetics and genomics during the 5-year period 1991-1995. Nonetheless, both of my Nobel number distributions are flat for the time
periods investigated. My flat distributions imply, moreover, that the $N_{P}(t)$ distributions of the examined Nobel laureates in Physiology and Medicine and the $N_{M}(t)$ distributions of the examined Nobel laureates in Physics are also, on average, flat over the time frames investigated.

A potential explanation for these flat distributions can be found in Figs. 1 and 3 for physics and in Fields (2014; 2015a) for biomedicine: Nobel laureates in both disciplines are richly connected both intergenerationally and between subdisciplines. They do not, in other words, form exclusive generational or subdisciplinary cliques. The limiting case of such rich inter-laureate co-authorship connections is the one in which every laureate during a given period is directly connected to every other laureate; the co-authorship subgraph of Nobel laureates is then a complete graph $K_{N}$ for some number $N$ of laureates. In this case, an arbitrarily-selected non-laureate located $l$ steps from a single closest laureate would be connected to every other laureate by a path of $l+1$ steps, as shown in Fig. 9a. Relaxing the completeness constraint in a way that is random in time - corresponding to no statistically-significant generational or subdisciplinary cliques - produces a flat dispersion around a flat mean. For example, if the probabilities that any two selected laureates are separated by one, two or three co-authorship steps are equal, a Nobel number distribution such as the one shown in Fig. 9b is produced. Introducing additional, equally-short paths from an arbitrary non-laureate to the collection of laureates overlays multiple such random distributions, with a result that is flat as in the example of Fig. 2. Cosmologist Stephen Hawking, for example, is two co-authorship steps from Murray GellMann (see Fig. 1) via James Hartle and hence three steps from David Pines, as well as three steps from John Wheeler (see Fig. 1) via Hartle and Kip Thorne. His Nobel number distribution can, therefore, never be more than three steps higher than Pines', and will be lower than Pines' only for those laureates closer to Gell-Mann or Wheeler than to Pines. This outcome is general: any physicist, and indeed any researcher in any discipline who is included within the giant component of the co-authorship graph, i.e. who has a co-authorship path of finite length to any of the researchers shown in Figs. 1, 3, or 8 or to Erdős, has at least one closest Nobel laureate in Physics. The Nobel number distribution for physics of any such researcher, therefore, can be expected to have the form of Fig. 9b for some value of $l$.


Fig. 9: A simple model can explain flat Nobel number distributions. a) If all $N$ laureates in a given period form a complete co-authorship graph $K_{N}$, a non-laureate located $l$ steps from a single closest laureate, here assumed to be in 1975, will be $l+1$ steps from all others. b) If minimal path lengths between laureates are randomly distributed within a fixed range, a distribution with flat dispersion around a flat mean is produced. A connection of $l$ steps to a single closest laureate in 1975 is assumed as in a) above.

If this explanation is correct, the flat Nobel number distributions in Figs. 2, 5 and 6 indicate that the samples of co-authorship connections between Nobel laureates in Physics or in Physiology or Medicine
shown in Figs. 1, 3 and 8 are representative, not exceptional. Arbitrarily-selected laureates can be expected to have other laureates nearby, and these nearby laureates may represent different generations or subdisciplines. Figure 4a shows that, with few exceptions, this expectation is born out for Physics laureates between 1951 and 2000; virtually all are within 5 co-authorship steps of some laureate shown in Fig. 1 and hence are within 6 or 7 steps of several others. The results shown in Fields (2015a) confirm this expectation for Physiology and Medicine laureates between 1991 and 2010. It will be interesting to see whether this pattern of dense co-authorship connections appears among Nobel laureates in Chemistry or Economics, two disciplines in which, unlike physics or biomedicine, "big science" collaborations are still rare. That this pattern characterizes Turing Award and von Neumann Medal winners in computer science, another "small science" discipline, is shown in Fields (in press).

## Conclusion

While citation-based measures dominate both academic assessment procedures (e.g. Gläser and Laudel, 2007) and maps of science (e.g. Moya-Anegón et al., 2007; Klavans and Boyack, 2009; Rafols, Porter and Leydesdorff, 2010; Boyack and Klavans, 2014), co-authorship graphs provide not just unique windows into the social and lineage structures of research communities but also an opportunity to investigate the "shapes" of both disciplines and the boundaries between disciplines at resolutions down to the level of individual researchers. Co-authorship graphs that include Nobel laureates provide, in particular, a means to visualize the "centers" of disciplines as informally defined by recognized scientific achievement. Nobel numbers complement time-independent co-authorship graphs by providing a way of tracking, with respect to arbitrarily-selected reference points, the trajectories of such centers as they move through the co-authorship graph as a function of time.

What has been shown here is that the "center" of physics as defined by Nobel laureates from 1951 to 2000 is remarkably stable: despite occasional excursions into distant and, at least in 2000, sparsely populated areas of the co-authorship graph, the center returns to the same region often enough that both the decade-to-decade means and the dispersions of Nobel number distributions computed from several reference points, including some outside of physics, remain roughly flat. Even for the most sensitive reference tested, all of Fig. 1 considered as a single point, the decade-to-decade average Nobel number increases by less than two co-authorship steps from the 1950s to the 1990s. As Fig. 1 includes Nobel laureates in Physics back to 1921, this result suggests that the co-authorship center of physics hardly moved during most of the $20^{\text {th }}$ century, despite dramatic changes in concepts, technology, funding for research and numbers of physicists. As Marie Curie (Physics, 1903 and Chemistry, 1911) is separated from James Chadwick and thus Fig. 1 by only two co-authorship steps (via Earnest Rutherford, Chemistry, 1908), this relative stability of the center may characterize the entire history of Nobel Prizes in Physics.

The present study raises several questions about the social structure of scientific disciplines generally. First, obviously, is the question of whether Nobel laureates provide a reasonable marker for the intuitive "centers" of disciplines as has been assumed here. High citation counts, directorships of major institutes or laboratories and influence on the flow of research funding are also intuitivelyappealing indicators of centrality. Some Nobel laureates are highly cited, administratively powerful and financially influential, but many individuals with these characteristics are not Nobel laureates; indeed the ISI "Highly Cited Researchers 2001" list, generated from papers published between 1981 and 1999, includes only four Nobel laureates from the cohort studied here, J. Georg Bednorz (1987),

Karl Müller (1987), Horst Störmer (1998) and Daniel Tsui (1998). ${ }^{4}$ While one might expect most if not all highly cited or otherwise influential individuals to have low Nobel numbers (or the equivalents) within their disciplines as do Teller and Pines, this has yet to be documented in more than a few specific cases. Second, how mobile are disciplinary centers? Is the motion of the center over time highly dependent on how the center is defined, or do all intuitively-reasonable definitions of centrality yield roughly the same trajectories? Third, are Nobel laureates and other "central" individuals richly connected across generations and subdisciplines in other disciplines as they appear to be in physics, biomedicine and computer science (Fields, 2014; 2015a; in press)? If so, do these connections lead to long-term relative immobility of the center as they appear to do in physics? Answering these questions would provide an interesting complement to the inferences about the time dependence of disciplinary structures that are obtainable with time-dependent citation-based maps of science.

## Conflict of Interest statement

The author declares no financial or other conflicts of interest regarding the work reported here.

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## Appendix

Erdős paths of all Nobel laureates in Physics, 1951-2000.
Nobel laureates, with names in bold, are listed in sequence by award year. Upper limits on Erdős numbers conferred by the co-authorship paths shown are indicated in parentheses after names. Path termini marked by * may be found in https://files.oakland.edu/users/grossman/enp/Erdos2.html; (Erdős Number Project data files); path termini marked by ** appear in http://www.oakland.edu/enp/erdpaths/ (Nobel Prize in Physics or Other Distinguished Scholars lists). Note that some values on these lists as of December, 2014 are decreased here. Path termini marked by $\dagger$ may be found at http://chrisfieldsresearch.com/erdos.htm. Multiple Nobel laureates appear in some paths.

1951:
E. T. S. Walton (7) - J. D. Cockcroft (6) - C. D. Ellis (6) - J. Chadwick (5) - M. Goldhaber (4) - E. Teller (3) - N. Metropolis (2)*

1952:
F. Bloch (5) - N. E. Bradbury (4) - L. A. Young (3) - G. E. Uhlenbeck (2)*
E. M. Purcell (4) - N. F. Ramsey (3)**

1953:
F. Zernike (4) - C. van Lier (3) - G. E. Uhlenbeck (2)*

1954:
M. Born (3)**
W. Bothe (8) - H. Geiger (7) - E. Rutherford (6) - J. Chadwick (5) - M. Goldhaber (4) - E. Teller (3) - N. Metropolis (2)*

1955:
W. E. Lamb, Jr. (3)**
P. Kusch (5) - I. I. Rabi (4)**

1956:
J. Bardeen (5)**
W. Brattain (6)**
W. Shockley (5) - J. R. Pierce (4) - C. E. Shannon (3)**

1957:
C. N. Yang (4) ${ }^{* *}$
T. D. Lee (5)**

1958:
P. A. Cherenkov (8) - E. G. Bessonov (7) - K.-S. Kim (6) - R. Schlueter (5) - E. Sugerbaker (4) - C. A. Fields (3) $\dagger$
I. Y. Tamm (8) - I. M. Frank (7) - V. L. Ginzburg (6) - L. Landau (5) - G. Gamow (4) - H. Bethe (3)**

1959:
E. Segrè (4) ${ }^{* *}$
O. Chamberlain (5)**

1960:
D. A. Glaser (5) - L. H. Thompson (4) - A. V. Carrano (3) - M. S. Waterman (2)*

1961:
R. Hofstadter (5)**
R. L. Mössbauer (5) - D. H. Sharp (4) - J. A. Wheeler (3) ** 1962:
L. Landau (5) - G. Gamow (4) - H. Bethe (3)**

1963:

## E. P. Wigner (3)**

J. H. D. Jensen (5) - M. Goeppert-Mayer (4) - M. Born (3)**

1964:
C. H. Townes (6) - J. Bardeen (5)**
N. Basov (7) - A. M. Prokharov (6) - C. K. Rhodes (5) - M. Scully (4) - W. E. Lamb, Jr. (3)** 1965:
R. P. Feynman (3)**

## J. Schwinger (4)**

S. I. Tomonaga (5) - J. R. Oppenheimer (4) - H. A. Bethe (3)**

1966:
A. Kastler (7) - J. Brossel (6) - F. Bitter (5) - D. E. Nagle (4) - E. Fermi (3)** 1967:

## H. A. Bethe (3)**

1968:
L. W. Alvarez (5)**

1969:
M. Gell-Mann (3)**

1970:
H. Alfvén (4) - E. Teller (3) - N. Metropolis (2)*
L. E. F. Néel (9) - P. Brissonneau (8) - M. Schlenker (7) - D. M. Goldberger (6) - B. Margolis (5) - V. Weisskopf (4) - E. Teller (3) - N. Metropolis (2)*

1971:
D. Gabor (8) - G. W. Stroke (7) - D. G. Falconer (6) - J. C. Vander Velde (5) - M. Goldhaber (4) - E. Teller (3) - N. Metropolis (2)*

1972:
J. Bardeen (5)**
L. N. Cooper (5) ${ }^{* *}$
J. R. Schrieffer (4)**

1973:
I. Giaever (7) - J. C. Fisher (6) - D. Turnbull (5) - F. Steitz (4) - E. P. Wigner (3)**

L Esaki (9) - P. J. Stiles (8) - E. H. Sondheimer (7) - A. H. Wilson (6) - J. Bardeen (5)**
B. D. Josephson (7) - J. Lekner (6) - A. R. Bishop (5) - J. R. Schrieffer (4)**
M. Ryle (9) - A. Hewish (8) - A. C. S. Readhead (7) - D. Pogosyan (6) - G. Smoot (5) - K. S. Babu (4) - F. Wilczek (3)**

1975:
A. Bohr (5)**
B. Mottelson (5)**
L. Rainwater (4) - I. I. Rabi (5)**

1976:
S. Ting (5) - V. Telegdi (4) - M. Gell-Mann (3)**
B. Richter (6) - J. Kadyk (5) - G. Goldhaber (4) - A. Pais (3) - G. E. Uhlenbeck (2)* 1977:
P. W. Anderson (5) - B. I. Halperin (4) - F. Wilczek (3)**
J. H. van Vleck (5) - V. Weisskopf (4) - E. P. Wigner (3)**
N. F. Mott (6) - H. Frohlich (5) - F. Steitz (4) - E. P. Wigner (3)**

1978:
P. L. Kapitza (5) - P. A. M. Dirac (4)**
A. Penzias (7) - P. M. Solomon (6) - R. McMahon (5) - G. Goldhaber (4) - A. Pais (3) - G. E. Uhlenbeck (2)*
R. Wilson (7) - P. M. Solomon (6) - R. McMahon (5) - G. Goldhaber (4) - A. Pais (3) - G. E. Uhlenbeck (2)*

1979:
S. L. Glashow (2)*
A. Salam (3)**
S. Weinberg (3)**

1980:
J. W. Cronin (6) - V. L. Fitch (5) - W. K. H. Panofsky (4) - H. A. Bethe (3)**

1981:

## A. L. Schawlow (5)**

K. Siegbahn (8) - D. W. Preston (7) - E. P. Chamberlin (6) - N. S. P. King (5) - J. J. Kraushaar (4) $\dagger$
N. Bloembergen (9) - W. C. Dickinson (8) - J. E. Brolley, Jr. (7) - L. Rosen (6) - E. R. Flynn (5) - J. J. Kraushaar (4) $\dagger$

1982:
K. G. Wilson (6) - B. Svetitsky (5) - G. Baym (4) - H. A. Bethe (3)**

1983:
S. Chandrasekhar (4)**
W. A. Fowler (5) - R. V. Wagoner (4) - D. N. Schramm (3) - S. L. Glashow (2)*

1984:
S. van der Meer (6) - C. Rubbia (5) - V. Telegdi (4) - M. Gell-Mann (3)**

1985:
K. von Klitzing (5)**

1986:
G. Binnig (6)**
H. Rohrer (7)**
E. Ruska (8) - H. Ruska (7) - D. H. Moore (6) - E. Y. Lasfargue (5) - H. Varmus (4) - F. S. Collins (3)**

1987:
J. G. Bednorz (10) - K. A. Müller (9) - K. W. Blazey (8) - F. H. Holzberg (7) - M. A. Kirk (6) - L. R. Greenwood (5) - J. R. Erskine (4) - R. H. Siemssen (3) - E. H. L. Aarts (2)*

1988:
M. Schwartz (7) - L. Lederman (6) - R. Jeppeson (5) - K. H. Hicks (4) - C. A. Fields (3) $\dagger$
J. Steinberger (5) - W. K. H. Panofsky (4) - H. A. Bethe (3)**

1989:

## N. F. Ramsey (3)**

H. G. Dehmelt (7) - G. Gabrielse (6) - L. S. Brown (5) - J. N. Bahcall (4) - H. A. Bethe (3)**
W. Paul (8) - H. G. Bennewitz (7) - D. R. Hamilton (6) - W. P. Alford (5) - R. A. Emigh (4) - C. A. Fields (3) $\dagger$

1990:
R. E. Taylor (5) - W. K. H. Panofsky (4) - H. A. Bethe (3)**
J. Friedman (5) - W. K. H. Panofsky (4) - H. A. Bethe (3)**
H. W. Kendall (5) - W. K. H. Panofsky (4) - H. A. Bethe (3)**

1991:
P.-G. De Gennes (6) - P. Nozières (5) - D. Pines (4) - M. Gell-Mann (3)**

1992:
G. Charpak (3) - S. L. Glashow (2)*

1993:
R. A. Hulse (5) - J. H. Taylor (4)**

1994:
C. G. Shull (6) - A. Zeilinger (5) - P. Zoller (4) - F. Wilczek (3)**
B. N. Brockhouse (8) - J. M. Rowe (7) - J. J. Rush (6) - W. W. Havens, Jr. (5) - I. I. Rabi (4)** 1995:
F. Reines (5) - M. Goldhaber (4) - E. Teller (3) - N. Metropolis (2)* M. L. Perl (5) - Goldhaber (4) - A. Pais (3) - G. E. Uhlenbeck (2)* 1996:
D. D. Osheroff (5)**
D. M. Lee (6)**
R. C. Richardson (6)**

1997:
S. Chu (6) - D. Herschlag (5) - O. Uhlenbeck (4) - G. D. Stormo (3) - A. Ehrenfeucht (2)* C. Cohen-Tannoudji (6) - W. D. Phillips (5) - P. Zoller (4) - F. Wilczek (3)** 1998:
R. B. Laughlin (5) - D. Pines (4) - M. Gell-Mann (3)**
H. L. Störmer (7) - W. Wiegmann (6) - C. A. Lee (5) - I. I. Rabi (4)**
D. C. Tsui (7) - W. Wiegmann (6) - C. A. Lee (5) - I. I. Rabi (4)**

1999:
M. J. G. Veltman (6) - G. 't Hooft (5) - R. Jackiw (4) - H. D. Politzer (3) - S. L. Glashow (2)* 2000:
H. Kroemer (9) - J. H. English (8) - H. L. Störmer (7) - W. Wiegmann (6) - C. A. Lee (5) - I. I. Rabi (4)**
Z. I. Alferov (9) - D. Bimberg (8) - H. L. Störmer (7) - W. Wiegmann (6) - C. A. Lee (5) - I. I. Rabi (4)**
J. S. Kilby (12) - E. Keonjian (11) - J. J. Suran (10) - W. F. Chow (9) - D. A. Paynter (8) - C. A. Guderjahn (7) - R. W. Boom (6) - J. R. Richardson (5) - R. R. Wilson (4) - H. A. Bethe (3)**


[^0]:    3 Citation searches conducted February, 2015.

